Glass phase of randomly polymerized membranes

Youngah Park and Chulan Kwon
Department of Physics, Myoung Ji University, Yongin, Kyonggi-do, Seoul 449-728, Korea
(Received 23 February 1996)

Polymerized membranes with quenched disorder in the internal metric are studied in the mean-field approximation. From the stability analysis of the replica-symmetric solution, we find a mixed (flat-glass) phase. This phase is the frozen phase with a nonzero average tangent field but with broken replica symmetry. This phase may correspond to the observed wrinkled phase of a partially polymerized membrane.
[S1063-651X(96)03708-7]

PACS number(s): 87.22.Bt, 64.70.Pf

Polymerized membranes, which are two-dimensional generalizations of linear polymer chains [1,2], have been studied intensively during the last few years. Unlike linear polymers, polymerized membranes with fixed connectivity and nonzero shear modulus are predicted to exhibit both the high temperature crumpled phase and the low temp temperature flat phase. The crumpled phase has been observed in simulations on non-self-avoiding phantom membranes [3–5] and has also been recently seen in Monte Carlo simulations of self-avoiding tethered surfaces modeled by impenetrable flexible plaquette [6]. The nonzero temperature flat phase is described by an anomalous elasticity with wave vector dependent elastic moduli that vanish and a bending rigidity that diverges at a long wavelength.

Recent studies on the polymerized membranes are concentrated on the effects of internal disorder. Recently Mutz, Bensimon, and Breinne observed that upon cooling partially polymerized vesicles undergo a spontaneous transition to a wrinkled rigid structure [7]. It was suggested that this transition could be a membrane equivalent of the spin-glass transition. Motivated by this experiment, there have been intensive theoretical investigations about the effects of disorder.

Recently Nelson and Radzihovsky [8] considered the effects of quenched random impurities leading to random disorder in the preferred metric tensor. They found that the flat phase remains stable with respect to such randomness at $T>0$ but becomes unstable at $T=0$ because of the disorder-induced softening of the bending rigidity. Morse, Lubensky, and Grest found [9] that using the ϵ expansion the presence of impurity-induced random curvature leads to a disordered flat phase stable at $T=0$ but unstable to temperature. Simultaneously with Morse, Lubensky, and Grest [9], Le Doussal and Radzihovsky found several flat-glass phases stable at $T>0$ using the self-consistent calculation [10]. Radzihovsky and Le Doussal [11] predicted that the flat phase at $T>0$ becomes unstable toward a crumpled glass phase similar to a spin glass at large enough (nonperturbative) disorder strength. Here a crumpled spin-glass phase is characterized by a vanishing average tangent field $[\langle \partial_\alpha r_i \rangle]$ but with a nonzero Edwards-Anderson spin-glass order parameter $[\langle \partial_\alpha r_i \rangle \langle \partial_\beta r_j \rangle]$. By investigating the effects of long-range disorder on the flat phase, Attal, Chaieb, and Bensimon [12] studied a mean-field model of a membrane with quenched random curvature. They find the replica-symmetry-breaking (RSB) solution and the instability line separating a flat phase from a new mixed phase with broken ergodicity.

In this paper, we investigate the possibility that the polymerized membrane with quenched disorder can exhibit a flat-glassy phase characterized by a nonzero average tangent field $[\langle \partial_\alpha r_i \rangle]$ and a nonzero spin-glass order parameter $[\langle \partial_\alpha r_i \rangle \langle \partial_\beta r_j \rangle]$ with broken replica symmetry within the approximation of the mean-field theory.

We use the disordered Hamiltonian with the quenched random disorder in the preferred metric, as was proposed in Refs. [8] and [10]. The probability of a particular configuration for fixed disorder configuration is proportional to $\exp[-H(\vec{r})/T]$ where

$$H=\int d^D x \Big(\tfrac{1}{2}\kappa|\nabla^2 \vec{r}(\vec{x})|^2+\tfrac{1}{4}\mu\{\partial_\alpha \vec{r}\cdot\partial_\beta \vec{r} - \delta_{\alpha\beta}[1+2\,\delta c(\vec{x})]\}^2+\tfrac{1}{8}\lambda\{\partial_\alpha \vec{r}\cdot\partial_\alpha \vec{r} - D[1+2\,\delta c(\vec{x})]\}^2\Big). \tag{1}$$

In the above, the surface configuration is described by $\vec{r}(\vec{x})$, where $\vec{x}$ is a D-dimensional internal coordinate labeling the membrane monomers, and $\vec{r}$ describes the embedding in a d-dimensional space. Here κ is the bending rigidity, and μ and λ are the elastic Lame coefficients of the membrane. limit We take $\delta c(x)$ to be a zero mean Gaussian quenched random field with probability distribution $P[\delta c(x)] \propto \exp[-1/2\sigma \int d^D x \delta^2 c(x)]$. Then the field $\delta c(x)$ describes random dilutions and compressions in the local preferred metric, $\delta_\alpha \vec{r}\cdot\delta_\beta \vec{r}=\delta_{\alpha\beta}[1+2\,\delta c(x)]$, due to disorder. The partition function for each configuration of disorder is given by

$$Z=\int D\vec{r}\exp\left(-\frac{1}{T}H\right). \tag{2}$$

Then the disorder averaged free energy is

$$F=-T[\ln Z], \tag{3}$$

where the square bracket [ ] refers to the average over the quenched disorder δc with the weight $P[\delta c]$.

In what follows, we use the replica formalism to treat the effect of disorder. The disorder averaged free energy F is obtained from the replicated, disorder averaged partition function $[Z^n]$ by the analytic continuation of all the results to $n\to 0$, making use of the identity

$$F=-T[\ln Z]=-T\lim_{n\to 0}\frac{\ln[Z^n]-1}{n}. \tag{4}$$

Performing the average of Z^n, we obtain

$$[Z^n]=\int\prod_{a=1}^{n}D\vec{r}_a\exp\Bigg\{-\beta\int d^Dx\sum_{a=1}^{n}\left[\tfrac{1}{2}\kappa|\nabla^2\vec{r}(\vec{x})|^2+\tfrac{1}{4}\mu(\partial_\alpha\vec{r}_a\cdot\partial_\beta\vec{r}_a-\delta_{\alpha\beta})^2+\tfrac{1}{8}\lambda(\partial_\alpha\vec{r}_a\cdot\partial_\alpha\vec{r}_a-D)^2\right]+\tfrac{1}{8}\beta^2\sigma\int d^Dx\sum_{\alpha\beta ab}(\partial_\alpha\vec{r}_a\cdot\partial_\alpha\vec{r}_a)(\partial_\beta\vec{r}_b\cdot\partial_\beta\vec{r}_b)\Bigg\}, \tag{5}$$

where the redefinition $(\lambda D+2\mu)^2\sigma\to\sigma$ is made and $\beta=1/T$. In the above we ignored $O(n^2)$ terms which will vanish in the limit $n\to 0$. Following the derivation of Radzihovsky and Le Doussal [11], we introduce auxiliary fields $\chi^a_{\alpha\beta}$ and $Q^{ab}_{\alpha\beta ij}$ and perform the Hubbard transformations on the quartic terms of the disorder averaged Hamiltonian. We obtain

$$[Z^n]=\int D\vec{r}_aD\chi^a_{\alpha\beta}DQ^{ab}_{\alpha\beta ij}\exp\Bigg\{-\beta\sum_{a=1}^{n}\int d^Dx\Bigg[\frac{1}{2}\kappa|\nabla^2\vec{r}(\vec{x})|^2-\frac{\alpha}{2}(\chi^a_{\alpha\beta})^2-\frac{\gamma}{2}(\chi^a_{\alpha\alpha})^2+\frac{1}{2}\chi^a_{\alpha\beta}(\partial_\alpha\vec{r}_a\cdot\partial_\beta\vec{r}_a-\delta_{\alpha\beta})-\frac{1}{4}\beta\sigma D(\partial_\alpha\vec{r}_a\cdot\partial_\alpha\vec{r}_a)\Bigg]+\int d^Dx\sum_{a\neq b}\Bigg[-\frac{1}{8}\beta^2\sigma(Q^{ab}_{\alpha\beta ij})^2+\frac{1}{4}\beta^2\sigma Q^{ab}_{\alpha\beta ij}\partial_\alpha r^i_a\partial_\beta r^j_b\Bigg]\Bigg\}, \tag{6}$$

where $\alpha=1/2\mu$ and $\gamma=(\lambda-\beta\sigma)/(2\mu[2\mu+D(\lambda-\beta\sigma)])$.

The free energy can now be computed in the following way. We split $\vec{r}_a(\vec{x})$ into its average and fluctuations

$$\vec{r}_a(\vec{x})=\vec{r}_0+\delta\vec{r}_a(\vec{x}), \tag{7}$$

where $\vec{r}_0=\langle\vec{r}_a\rangle$, $\delta\vec{r}=\vec{r}_a-\vec{r}_0$ and we perform the Gaussian integration over $\delta\vec{r}_a$. Within the approximation of the mean-field theory, we now perform the remaining integrals over $\chi^a_{\alpha\beta}$ and $Q^{ab}_{\alpha\beta ij}$ using the saddle point method, which is valid for $d\gg D\gg 1$. We assume the saddle point solution has the following form:

$$\chi^a_{\alpha\beta}=\chi_a\delta_{\alpha\beta}, \tag{8}$$

$$Q^{ab}_{\alpha\beta ij}=q_{ab}\delta_{\alpha\beta}\delta_{ij}(1-\delta_{ab}), \tag{9}$$

$$\vec{r}_0=\zeta x^\alpha\vec{e}_\alpha, \tag{10}$$

where $\vec{e}_\alpha$ is a unit vector.

Now we find

$$\frac{Fn}{L^D}=\left\{\frac{D}{2}(\zeta^2-1)\sum_a\chi_a-\frac{D}{2}(\alpha+\gamma D)\sum_a\chi_a^2-\frac{1}{4T}\sigma D^2\zeta^2n+G_0(\mathbf{q})\right\}_{\text{s.p.}} \tag{11}$$

where the notation s.p. means that one has to evaluate the expression within the bracket for saddle point values with respect to χ_a, ζ, and q_{ab} and

$$G_0(\mathbf{q})=\frac{1}{8T}\sigma D\left(\sum_{a\neq b}q^2_{ab}d-2\sum_{a\neq b}q_{ab}\zeta^2\right)+\frac{1}{2}Td\int\frac{d^Dk}{(2\pi)^D}\ln\det\mathbf{q}. \tag{12}$$

In the above the matrix element of $\mathbf{q}$ are defined as

$$(\mathbf{q})_{ab}=q_{ab}\quad(a\neq b),$$

$$(\mathbf{q})_{aa}=\frac{\kappa k^2+\chi_a-\dfrac{\sigma D}{2T}}{-\dfrac{\sigma}{2T}}. \tag{13}$$

The saddle point equation for q_{ab} derived from G_0 reads, in the $n\to 0$ limit [13]

$$\frac{1}{4T}\sigma D(q^2_{ab}d-2q_{ab}\zeta^2)+\frac{Td}{2}\int\frac{d^Dk}{(2\pi)^D}(\mathbf{q}^{-1})_{ab}=0. \tag{14}$$

Following Radzihovsky and Le Doussal, we use a a replica symmetry(RS) ansatz

$$q_{ab}=q(1-\delta_{ab}),\quad \chi_a=\chi. \tag{15}$$

Then the RS saddle point equations become

$$\frac{\zeta^2}{d}=q\left(1-\frac{\sigma}{2D}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\left(\kappa k^2+\chi-\dfrac{\sigma D}{2T}+\dfrac{\sigma q}{2T}\right)^2}\right), \tag{16}$$

$$1-\zeta^2+2\chi(\alpha+\gamma D)=\frac{Td}{2D}\int\frac{d^Dk}{(2\pi)^D}\left(\frac{2}{\kappa k^2+\chi-\dfrac{\sigma D}{2T}+\dfrac{\sigma q}{2T}}+\frac{q\sigma/T}{\left(\kappa k^2+\chi-\dfrac{\sigma D}{2T}+\dfrac{\sigma q}{2T}\right)^2}\right), \tag{17}$$

$$D\zeta\left(\chi+\frac{\sigma q}{2T}-\frac{\sigma D}{2T}\right)=0. \tag{18}$$

By analyzing the saddle point equations, it was shown that there are three phases as follows [10]:

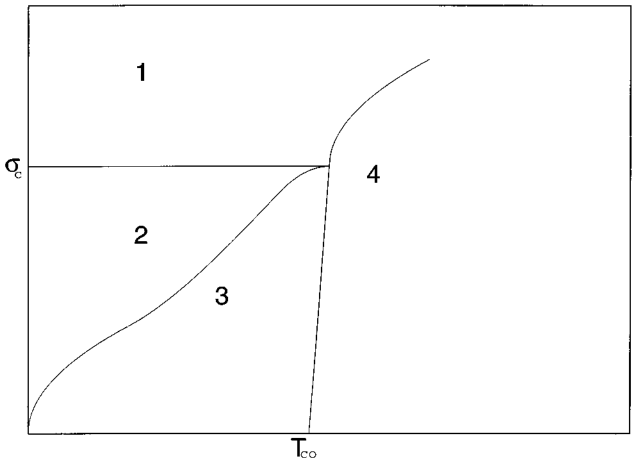

FIG. 1. Mean-field phase diagram for $D>4$.

(1) The crumpled phase described by $\zeta=0$, and $q=0$ which corresponds to region 4 of Fig. 1.

(2) The flat phase described by $q\neq 0$ and $\zeta\neq 0$, which corresponds to regions 2 and 3 of Fig. 1.

(3) The crumpled spin-glass phase described by $q\neq 0$ and $\zeta=0$, which corresponds to region 1 of Fig. 1.

The transition from flat to the crumpled spin-glass phase occurs at $\sigma=\sigma_c$ where

$$\sigma_c{}^{-1}=\frac{1}{2D}\int \frac{d^Dk}{(2\pi)^D}\,\frac{1}{\kappa^2k^4}. \tag{19}$$

Also the transition from the flat to the crumpled phase occurs at $T=T_c(\sigma)$ where

$$T_c(\sigma)=\tfrac{1}{2}T_{c0}\left(1+\sqrt{1+4\sigma D(\alpha+\gamma D)T_{c0}{}^{-2}}\right) \tag{20}$$

with

$$T_{c0}=\frac{d}{D}\int \frac{d^Dk}{(2\pi)^D}\,\frac{1}{\kappa k^2}. \tag{21}$$

We note that our $T_c(\sigma)$ differs from that of Ref. [10].

We now investigate the stability of the above RS solution to find a mixed phase with a broken replica symmetry. To examine the question that Eqs. (16)–(18) give a maximum of the free energy expression Eq. (11), we write

$$\chi_a=\chi+\delta q_{aa}, \quad q_{ab}=q+\delta q_{ab} \quad (a\neq b) \tag{22}$$

where χ and q are the RS saddle point values described by Eqs. (15)–(18). Then the second order variation of the free energy with respect to δq_{aa} and δq_{ab} gives

$$2n\delta^2 f=-D(\alpha+\gamma D)\sum_a (\delta q_{aa})^2+\frac{1}{4T}\sigma Dd\sum_{a\neq b}(\delta q_{ab})^2 -\frac{d}{2\beta}\int \frac{d^Dk}{(2\pi)^D}Tr(\mathbf{q}^{-1}\delta\mathbf{q})^2 \tag{23}$$

$\delta q_{aa}=0$ by construction. Now we find

$$(\mathbf{q}^{-1})_{ab}=u\,\delta_{ab}+v, \tag{24}$$

where

$$u=\frac{1}{q_{aa}-q}, \quad v=\frac{-q}{(q_{aa}^2-q)^2}. \tag{25}$$

The matrix G associated with the above quadratic form has seven different types of matrix elements [14]. The coefficients of the δq_{aa} alone are given by

$$G_{aa}=\frac{\partial^2 f}{\partial(\delta q_{aa})\partial(\delta q_{aa})} =-(\alpha+\gamma D)-\frac{d}{2\beta}\left(-\frac{\beta\sigma}{2}\right)^{-2} \times\int \frac{d^Dk}{(2\pi)^D}(u^2+2uv+v^2)=A, \tag{26}$$

$$G_{ab}=\frac{\partial^2 f}{\partial(\delta q_{bb})\partial(\delta q_{aa})}=-\frac{d}{2\beta}\left(-\frac{\beta\sigma}{2}\right)^{-2}\int \frac{d^Dk}{(2\pi)^D}v^2 =B. \tag{27}$$

The coefficients of the $\delta q_{ab}(a\neq b)$ alone have the forms

$$G_{(ab)(ab)}=\frac{\partial^2 f}{\partial(\delta q_{ab})\partial(\delta q_{ab})} =\frac{1}{4}\beta\sigma Dd-\frac{d}{2\beta}\int \frac{d^Dk}{(2\pi)^D}(u^2+2uv+v^2) =P, \tag{28}$$

$$G_{(ab)(ac)}=\frac{\partial^2 f}{\partial(\delta q_{ab})\partial(\delta q_{ac})}=-\frac{d}{2\beta}\int \frac{d^Dk}{(2\pi)^D}(v^2+uv)=Q, \tag{29}$$

$$G_{(ab)(cd)}=\frac{\partial^2 f}{\partial(\delta q_{ab})\partial(\delta q_{cd})}=-\frac{d}{2\beta}\int \frac{d^Dk}{(2\pi)^D}v^2 =R. \tag{30}$$

The coefficients of the cross terms have the forms

$$G_{a(ab)}=\frac{\partial^2 f}{\partial(\delta q_{aa})\partial(\delta q_{ab})} =-\frac{d}{2\beta}\left(-\frac{\beta\sigma}{2}\right)^{-1}\int \frac{d^Dk}{(2\pi)^D}(2uv+v^2) =C, \tag{31}$$

$$G_{a(bc)}=\frac{\partial^2 f}{\partial(\delta q_{aa})\partial(\delta q_{bc})}=-\frac{d}{2\beta}\left(-\frac{\beta\sigma}{2}\right)^{-1}\int \frac{d^Dk}{(2\pi)^D}v^2 =D. \tag{32}$$

There are at most five distinct eigenvalues of the matrix.

(1) The first is

$$\lambda_1=P-2Q+R \tag{33}$$

and corresponds to the eigenvector for which

$$\sum_b \delta q_{ab}=0 \quad \text{for all } a. \tag{34}$$

(2) The second and the third are

$$\lambda_{2\pm}=\tfrac{1}{2}[A+(n-1)B+P+2(n-2)Q+\tfrac{1}{2}(n-2)(n-3)R] \pm\tfrac{1}{2}\{[A+(n-1)B-P-2(n-2)Q -\tfrac{1}{2}(n-2)(n-3)R]^2+2(n-1) \times[2C+(n-2)D]^2\}^{1/2}, \tag{35}$$

corresponding to the eigenvectors

$$\sum_{ab} \delta q_{ab}=0, \quad \text{but} \quad \sum_b \delta q_{ab}\neq 0. \tag{36}$$

(3) Finally the fourth and the fifth are

$$\lambda_{4\pm}=\tfrac{1}{2}[A-B+P+(n-4)Q-(n-3)R] \pm\tfrac{1}{2}\{[A-B-P-(n-4)Q+(n-3)R]^2 +4(n-2)(C-D)^2\}^{1/2} \tag{37}$$

and correspond to eigenvectors such that $\Sigma_{ab}\delta q_{ab}\neq 0$. These five eigenvalues are distinct for general n, but for $n=0$, Eq. (35) and Eq. (37) coincide. Hence there are three distinct eigenvalues in the limit $n\to 0$. We find that the eigenvalue λ_1 is positive in the flat phase and the crumpled phase while it is zero in the crumpled spin-glass phase. For the eigenvalue $\lambda_{2\pm}$, we find that λ_{2+} remains positive in all regions but λ_{2-} changes sign from positive to negative as T decreases in the flat phase. In Fig. 1, the line separating flat regions 2 and 3 into two regions is the Almeida-Thouless instability line in which λ_{2-} changes the sign. The Almeida-Thouless instability line signals the transition from the RS flat phase to the frozen phase with a nonzero average tangent vector, but with broken replica symmetry. We call this mixed phase a flat-glass phase.

Near $\sigma=\sigma_c$, $T=T_c(\sigma,c)$, the transition is described by the line

$$T_c-T\propto\sqrt{(\sigma_c-\sigma)}. \tag{38}$$

Also near $T=0$ in the flat phase, the instability line equivalent to the Almeida-Thouless line is described by

$$T\propto\sigma^2, \tag{39}$$

which implies that the RS flat phase is unstable at $T=0$ to infinitesimal disorder. Morse, Lubensky, and Grest [9] and Le Doussal and Radzihovsky [10] also found that flat-glass phase at $T=0$, respectively, using the method of the ϵ expansion and the self-consistent calculation in the presence of the extrinsic curvature disorder. Nelson and Radzihovsky [8] also predict that the flat phase becomes unstable at $T=0$. The instability of the flat phase at $T=0$ is also predicted by Attal, Chaieb, and Bensimon [12].

To conclude, we consider the polymerized membrane with quenched disorder in the preferred metric. Via the stability analysis of the mean-field RS solution, we find a new mixed phase called the flat-glass phase. This phase is characterized by frozen local tangents with broken replica symmetry. The flat-glass phase found in this paper may also correspond to the observed wrinkled phase of partially polymerized membrane.

We would like to thank Professor Tom C. Lubensky for useful discussions. This work is supported in part by the Ministry of Education through the Basic Science Research Institute of Seoul National University and in part by KOSEF.

[1] See, e.g., *Statistical Mechanics of Membranes and Interfaces*, edited by D. R. Nelson, T. Pitan, and S. Weinberg (World Scientific, Singapore, 1988).
[2] Y. Kantor, M. Kantor, and D. R. Nelson, Phys. Rev. A **35**, 3056 (1987).
[3] Y. Kantor and D. R. Nelson, Phys. Rev. A **38**, 4020 (1987).
[4] F. F. Abraham, W. E. Rudge, and M. Plishke, Phys. Rev. Lett. **62**, 1757 (1989).
[5] F. F. Abraham and D. R. Nelson, Science 249, 393 (1990); J. Phys. (Paris) **51**, 2653 (1990).
[6] A. Baumgartner and W. Renz, J. Phys. (France) I **1**, 1549 (1991).
[7] M. Mutz, D. Bensimon, and M. J. Breinne, Phys. Rev. Lett. **36**, 415 (1976).
[8] D. R. Nelson and L. Radzihovsky, Europhys. Lett. **16**, 2634 (1988); L. Radzihovsky and D. R. Nelson, Phys. Rev. A **44**, 3525 (1991).
[9] D. C. Morse, T. C. Lubensky, and G. S. Grest, Phys. Rev A **45**, 2151 (1992).
[10] P. Le Doussal and L. Radzihovsky, Phys. Rev. Lett. **69**, 1209 (1992); Phys. Rev. B **48**, 3548 (1993).
[11] L. Radzihovsky and P. Le Doussal, J. Phys. (France) I **2**, 599 (1992).
[12] R. Attal, S. Chaieb, and D. Bensimon, Phys. Rev. E **48**, 2232 (1993).
[13] A. Crisanti and H.-J. Sommers, Z. Phys. B **87**, 341 (1992).
[14] J. R. L. de Almeida and D. J. Thouless, J. Phys. A **11**, 983, (1978).